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140. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Determine (any way) whether the Diophantine equation $\left(\frac{2x-1}{3}\right) = x^2 + y^2$ has any positive integer solutions.

Solution by JACOB WESTLUND, Ph. D., Purdue University.

In order that $\frac{2x-1}{3}$ shall be an integer we must have $x=2+3a$, where a is a positive integer. Hence $(1+2a)^2 = (2+3a)^2 + y^2$ or after a few reductions $y^2 = 8a^2 - 6a + 3(a^2 - 1)$.

If a is odd, this equation is impossible, since in that case y must be even and hence all the terms except $6a$ divisible by 4.

If a is even, we put the equation in the form $y^2 = 8a^2 + 3a(a-2) - 3$. This shows that y must be odd and $y+3$ divisible by 8. Hence, setting $y=2b+1$, $4b^2 + 4b + 4$ should be divisible by 8 or $b(b+1)+1$ divisible by 2, which is impossible. Hence the given equation has no positive integer solutions.

Also solved by A. H. Holmes.

No solution of 141 has yet been received.

AVERAGE AND PROBABILITY.

178. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Two random planes cut a given sphere. What is the chance that they intersect within the sphere?

I. Solution by HENRY HEATON, Belfield, N. D.

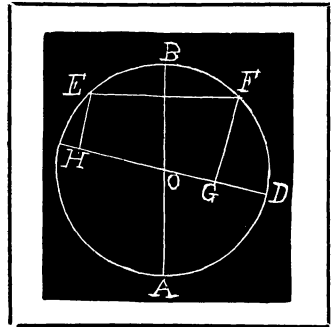
Let AB and CD be axes of the sphere perpendicular to the two planes and let EF be a trace of one of the planes.

Put $x=OI$, the distance of the plane through EF from the center of the sphere. Put $\theta = \angle BOC$. Then HG , the projection of EF upon $CD=2\sqrt{(a^2-x^2)}\sin\theta$.

It seems to be generally understood that the number of directions of the plane perpendicular to CD depends upon the number of different directions possible to CD , and that this depends upon the number of points in the surface of the sphere. Hence the number of planes of the direction θ is proportional to $\sin\theta$. The angle θ being supposed fixed the chance of intersection within the sphere is $\frac{HG}{CD} = \frac{\sqrt{(a^2-x^2)}\sin\theta}{a}$.

Hence the required probability is

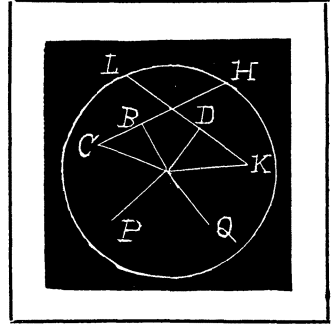
$$p = \int_0^{\frac{1}{2}\pi} \int_0^a \sqrt{(a^2-x^2)} \sin^2\theta d\theta dx / \int_0^{\frac{1}{2}\pi} \int_0^a a \sin\theta d\theta dx$$



$$= \frac{\pi}{4} \int_0^{\frac{1}{2}\pi} \sin^2 \theta d\theta / \int_0^{\frac{1}{2}\pi} \sin \theta d\theta = \frac{\pi^2}{16}.$$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let CH , LK be the diameters of the sections of the sphere made by the planes. B , D their centers; O the center of the sphere; OQ a line such that a line in the plane LK is parallel to the plane DOQ . $OC=OK=r$, $\angle COB=\theta$, $\angle KOD=\phi$, $\angle DOQ=\psi$. The limits of θ and ϕ are $\frac{1}{2}\pi$; of ψ , 0 and $\frac{1}{2}\pi$; of $\psi \pm (\theta-\phi)$ and $\theta+\phi$. The double sign is used $+$ for $\theta > \phi$, $-$ for $\theta < \phi$. Hence the chance p is



$$p = \int_0^{\frac{1}{2}\pi} \int_0^{\frac{1}{2}\pi} \int_{\pm(\theta-\phi)}^{\theta+\phi} d\theta d\phi d\psi / \int_0^{\frac{1}{2}\pi} \int_0^{\frac{1}{2}\pi} \int_0^{\pi} d\theta d\phi d\psi$$

$$= \frac{4}{\pi^3} \int_0^{\frac{1}{2}\pi} \int_0^{\frac{1}{2}\pi} \int_{\pm(\theta-\phi)}^{\theta+\phi} d\theta d\phi d\psi = \frac{8}{\pi^3} \int_0^{\frac{1}{2}\pi} \int_0^{\frac{1}{2}\pi} \phi d\theta d\phi = \frac{1}{\pi} \int_0^{\frac{1}{2}\pi} d\theta = \frac{1}{2}.$$

NOTE.—These two solutions differ in the method of distributing the direction of the random planes. ED. F.

NOTES AND NEWS.

Professor C. Alasia, mathematical editor of the "Rivista di Fisica e Matematica," of Pisa and Pavia, will review in that journal all new publications sent to him at Ozieri, Italy.

At the University of Chicago, Assistant Professor L. E. Dickson has been promoted to an associate professorship in mathematics, and Associate Professor Heinrich Maschke to a full professorship in mathematics.

The following courses in Mathematics and Mathematical Astronomy are to be given at the University of Chicago during the Summer Quarter of 1907 beginning June 15th: By Professor Moore: Graphical Methods in Algebra especially for teachers, 4 hours; Theory of Determinants, Advanced Course, 4 hours; General Seminar, 2 hours. By Professor Bolza: Theory of Functions of Complex Variables, 4 hours; Problems in Theory of Functions, 2 hours; Abelian Functions, 2 hours. By Assistant Professor Slaught: Integral Calculus, 5 hours; Differential Equations, 4 hours. By Associate Professor Dickson: Trigonometry, 5 hours; Solid Analytical Geometry, 5 hours; Continuous Groups, 4 hours. By Assistant Professor Moulton: Descriptive Astronomy, 5 hours; Introduction to Celestial Mechanics, 4 hours. By Assistant Professor Laves: Descriptive Astronomy, 5 hours; General Astronomy and Observatory Work, 5 hours. By Dr. Lunn: Curve Tracing and Differential Calculus, 5 hours; Dynamics of Oscillatory Systems, 4 hours. By Mr. Lennes: Plane Analytic Geometry, 5 hours; Critical Review of Secondary Mathematics, 4 hours. In the College of Education: By Professor Myers: Pedagogy of Elementary School Mathematics, Pedagogy of Secondary School Mathematics.